

Que: $x = 2 + \sqrt{3}$ and $x + y = 4$, then find the simplest value of $xy + \frac{1}{xy}$.

Solⁿ:

$$xy + \frac{1}{xy}$$
$$= \frac{x^2y^2 + 1}{xy}$$

given that

$$x = 2 + \sqrt{3}$$

$$x + y = 4 \Rightarrow y = 4 - x$$

$$= 4 - 2 - \sqrt{3}$$

$$= 2 - \sqrt{3}$$

$$\text{Again} = \frac{x^2y^2 + 1}{xy}$$

$$= \frac{1 + (2 + \sqrt{3})^2(2 - \sqrt{3})^2}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \frac{1 + \{(2 + \sqrt{3})(2 - \sqrt{3})\}^2}{(2 + \sqrt{3})(2 - \sqrt{3})}$$

$$= \frac{1 + \{(2)^2 - (\sqrt{3})^2\}^2}{(2)^2 - (\sqrt{3})^2}$$

$$\left[\because (a+b)(a-b) = a^2 - b^2 \right]$$

$$= \frac{1 + (4 - 3)^2}{4 - 3}$$

$$= 1 + 1^2 = 1 + 1 = 2 \text{ Ans:-}$$